

# Target mass corrections and twist-3 in the nucleon spin structure functions

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## Abstract

The Nachtmann moment is employed to study the contribution of twist-3 operator to the nucleon spin structure functions. Target mass corrections to the Cornwall-Norton moments of the spin structure functions  $g_{1,2}$  are discussed. It is found that the corrections play a sizeable role to the contribution of the twist-3  $\tilde{d}_2$  extracted from the Cornwall-Norton moments.

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## 1 Introduction

We know that the study of Bloom-Gilman quark-hadron duality is essential to understand the physics behind the connection between perturbative QCD (pQCD) and non-perturbative QCD [1]. In 2000, the new evidence of valence-like quark-hadron duality in the nucleon unpolarized structure functions  $F_2^{p,d}$  was reported by Jefferson Lab. [2]. The well-known Bloom-Gilman quark-hadron duality [3] tells that prominent resonances do not disappear relatively to the background even at a large  $Q^2$ . It also means that the average of the oscillate resonance peaks in the resonance region is the same as the scaling structure function at a large  $Q^2$  value. The origin of the Bloom-Gilman quark-hadron duality has been discussed by Rujula, Georgi and Politzer [4] with a QCD explanation. According to operator production expansion (OPE), it is argued that higher-twist effects turn to be small in the integral of the structure functions, and therefore, the leading-twist

plays a dominate role to the moments of the nucleon structure functions [4]. So far, the nucleon structure functions and the higher-twist effects have already been carefully and systemically studied [5]. Some detailed calculations for the higher-twist effects were carried out based on various theoretical approaches, like bag model [6], QCD sum rule [7-8], constituent quark model [9], Lattice QCD [10], and chiral soliton model [11]. Moreover, there are also several empirical analyses of the spin structure functions of  $g_1$  and  $g_2$  at low  $Q^2$ . The higher-twist effects, like the ones of the twist-3 and twist-4 terms, have been extracted from the data [12-16]. Those analyses can be more and more accurate because more and more precisely new measurements of the nucleon spin structure functions of  $g_1$  [17-19], and particularly of  $g_2$  [15,16,20], are available.

Usually, the contribution of the twist-3  $\tilde{d}_2$  is extracted from the measured  $g_{1,2}(x, Q^2)$  by calculating the moment of

$$I(Q^2) = \int_0^1 dx x^2 (2g_1(x, Q^2) + 3g_2(x, Q^2)) \rightarrow \tilde{d}_2(Q^2). \quad (1)$$

We know that the first moment of  $g_1$  can be generally expanded in inverse powers of  $Q^2$  in OPE [5]. It is

$$g_1^{(1)} = \int_0^1 dx g_1(x, Q^2) = \sum_{\tau=2, even}^{\infty} \frac{\mu_{\tau}(Q^2)}{Q^{\tau-2}} \quad (2)$$

with the coefficients  $\mu_{\tau}$  related to the nucleon matrix elements of the operators of twist  $\leq \tau$ . In eq. (2), the leading-twist (twist-2) component  $\mu_2$  is determined by the matrix elements of the axial vector operator  $\bar{\psi}\gamma_{\mu}\gamma_5\psi$ , summed over various quark flavors. The coefficient of  $1/Q^2$  term contains the contributions from the twist-2  $\tilde{a}_2$ , twist-3  $\tilde{d}_2$ , and twist-4  $\tilde{f}_2$ , respectively. Thus [5],

$$\mu_4 = \frac{1}{9}M^2(\tilde{a}_2 + 4\tilde{d}_2 + 4\tilde{f}_2), \quad (3)$$

where  $M$  is the nucleon mass. In eq. (3)  $\tilde{a}_2$  arises from the target mass corrections, and it is of purely kinematical origin. It relates to the third moment of the twist-2 part of  $g_1(x, Q^2; M=0)$  ( $2\tilde{a}_2 = \int_0^1 x^2 dx g_1(x, Q^2; M=0)$ ). The other higher-twist terms, like  $\tilde{d}_2$  and  $\tilde{f}_2$ , result from the reduced matrix elements, which are of the dynamical origin since they show the correlations among the partons [7,21]. If the contribution of the twist-3 term is well defined by eq. (1), then the contribution of the twist-4 term, like  $\tilde{f}_2$ , can be extracted according to eq. (3).

It should be mentioned that the method of eq. (1) to extract the twist-3 contribution  $\tilde{d}_2$  ignores the target mass corrections to  $g_{1,2}$ , since the relations

$$\int_0^1 x^2 g_1(x, Q^2) = \frac{1}{2} \tilde{a}_2, \quad \int_0^1 x^2 g_2(x, Q^2) = \frac{1}{3} (\tilde{d}_2 - \tilde{a}_2)$$

are used. In general, the nucleon target mass corrections should be considered completely in the studies of the nucleon structure functions [22], of the Bloom-Gilman quark-hadron duality [23-25], and of the Bjorken sum rule [26] at a moderate low  $Q^2 (\sim 1 - 5 \text{ GeV}^2)$ . We know that the target mass corrections to the nucleon structure functions are of the pure kinematical origin. They are different from the other higher-twist effects from dynamical multi-gluon exchanges or parton correlations. Before one can extract the higher-twist effects, it is important to remove the target mass corrections from the data [24]. There were several works about the target mass corrections to  $F_{1,2}(x, Q^2)$  and  $g_{1,2}(x, Q^2)$  in the literature [27-29]. Recently, the expressions of all the electromagnetic and electroweak nucleon spin structure functions with the target mass corrections have been explicitly given in Refs. [30-31].

In this work, in order to precisely extract the contribution of the twist-3 operators, the target mass corrections to eq. (1) will be discussed. In section 2, we explicitly give the target mass corrections to the integral  $I(Q^2)$ . Moreover, the advantage of the Nachtmann moments is stressed. In section 3, the numerical estimate of the target mass corrections to  $I(Q^2)$  is given comparing to the result of the Nachtmann moment. The last section is devoted for conclusions.

## 2 Twist-3 matrix elements and the target mass corrections

Here, we use the notations of Piccione and Ridolfi [30] for the spin structure functions and for their moments. We know that the well-known Cornwall-Norton (CN) moments are

$$g_{1,2}^{(n)}(Q^2) = \int_0^1 dx x^{n-1} g_{1,2}(x, Q^2). \quad (4)$$

In Refs. [30-31], the target mass corrections to the nucleon spin structure functions  $g_1$  and  $g_2$  are explicitly given in terms of the CN moments of the matrix elements of the twist-2 (leading-twist) operator and twist-3 one. Up to twist-3, the results are

$$g_1^{(n)}(Q^2) = a_n + y^2 \frac{n(n+1)}{(n+2)^2} (na_{n+2} + 4d_{n+2})$$

$$\begin{aligned}
& +y^4 \frac{n(n+1)(n+2)}{2(n+4)^2} (na_{n+4} + 8d_{n+4}) \\
& +y^6 \frac{n(n+1)(n+2)(n+3)}{6(n+6)^2} (na_{n+6} + 12d_{n+6}) + \mathcal{O}(y^8)
\end{aligned} \tag{5}$$

with  $y^2 = M^2/Q^2$ , and

$$\begin{aligned}
g_2^{(n)}(Q^2) &= \frac{n-1}{n} (d_n - a_n) + y^2 \frac{n(n-1)}{(n+2)^2} (nd_{n+2} - (n+1)a_{n+2}) \\
& +y^4 \frac{n(n-1)(n+1)}{2(n+4)^2} (nd_{n+4} - (n+2)a_{n+4}) \\
& +y^6 \frac{n(n-1)(n+1)(n+2)}{6(n+6)^2} (nd_{n+6} - (n+3)a_{n+6}) + \mathcal{O}(y^8).
\end{aligned} \tag{6}$$

In eqs. (5) and (6),  $a_n$  and  $d_n$  are the reduced hadron matrix elements of the irreducible Lorentz operators [29-30]:  $R_1^{\sigma\mu_1 \cdots \mu_{n-1}}$  and  $R_2^{\lambda\sigma\mu_1 \cdots \mu_{n-2}}$ . The matrix elements of the operators:  $R_1^{\sigma\mu_1 \cdots \mu_{n-1}}$  (twist-2) and  $R_2^{\lambda\sigma\mu_1 \cdots \mu_{n-2}}$  (twist-3) can be written as [29-30]

$$\langle p, s | R_1^{\sigma\mu_1 \cdots \mu_{n-1}} | p, s \rangle = -2Ma_n M_1^{\sigma\mu_1 \cdots \mu_{n-1}}, \tag{7}$$

and

$$\langle p, s | R_2^{\lambda\sigma\mu_1 \cdots \mu_{n-2}} | p, s \rangle = Md_n M_2^{\lambda\sigma\mu_1 \cdots \mu_{n-2}}, \tag{8}$$

where  $M_1^{\sigma\mu_1 \cdots \mu_{n-1}}$  is the general rank- $n$  symmetric tensor which can be formed with one spin four-vector  $s$  and  $n-1$  momentum four-vectors  $p$ , and  $M_2^{\lambda\sigma\mu_1 \cdots \mu_{n-2}}$  is antisymmetric in  $(\lambda, \sigma)$ , symmetric in all other indices. The two tensors must be traceless. A typical example for a twist-3 operator is

$$d_2 : \quad \bar{\psi} \gamma_{\{\alpha} \tilde{F}_{\beta\}} \gamma_{\gamma} \psi \quad \text{twist} - 3 \tag{9}$$

where  $\{\cdot \cdot \cdot\}$  denotes symmetrizing the indices and subtracting the trace, and  $\tilde{F}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta} F_{\gamma\delta}$  is the dual gluon field strength [6-7]. We notice that the contribution of the leading-twist terms has a relation to the quark distributions functions. However, the contributions of the higher-twist operators (like twist-3) have no partonic interpretation [31]. In eq. (5) if we fix  $n = 1$ , then,

$$g_1^{(1)} = a_1 + y^2 \frac{2}{9} (a_3 + 4d_3) + \cdots \tag{10}$$

Comparing to eqs. (2-3), we see that  $\tilde{a}_2 = 2a_3$  and  $\tilde{d}_2 = 2d_3$ . In eqs. (5) and (6) only the contributions of the leading-twist and twist-3 operators are considered.

From eqs. (1) and (4-6), we see that

$$\begin{aligned}
I(Q^2) = 2g_1^{(3)} + 3g_2^{(3)} &= \int_0^1 x^2 dx (2g_1(x, Q^2) + 3g_2(x, Q^2)) \\
&= 2(d_3 + 6y^2 d_5 + 12y^4 d_7 + 20y^6 d_9) + \mathcal{O}(y^8) \\
&= \tilde{d}_2 + 6y^2 \tilde{d}_4 + 12y^4 \tilde{d}_6 + 20y^6 \tilde{d}_8 + \mathcal{O}(y^8)
\end{aligned} \tag{11}$$

Clearly,  $I(Q^2)$  contains the target mass corrections. It has mixed contributions from other higher-spin and twist-3 terms except for  $d_3$ (or  $\tilde{d}_2$ ).

To get the twist-3 contribution of an operator with a definite spin, we need to consider the Nachtmann moments. According to Ref. [29], the two Nachtmann moments of the spin structure functions are defined as

$$\begin{aligned}
M_1^{(n)}(Q^2) &= \int_0^1 dx \frac{\xi^{n+1}}{x^2} \left\{ \left[ \frac{x}{\xi} - \frac{n^2}{(n+2)^2} y^2 x \xi \right] g_1(x, Q^2) \right. \\
&\quad \left. - y^2 x^2 \frac{4n}{n+2} g_2(x, Q^2) \right\}, \quad (n = 1, 3, 5, \dots),
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
M_2^{(n)}(Q^2) &= \int_0^1 dx \frac{\xi^{n+1}}{x^2} \left\{ \frac{x}{\xi} g_1(x, Q^2) \right. \\
&\quad \left. + \left[ \frac{n}{n-1} \frac{x^2}{\xi^2} - \frac{n}{n+1} y^2 x^2 \right] g_2(x, Q^2) \right\}, \quad (n = 3, 5, \dots),
\end{aligned} \tag{13}$$

where the Nachtmann variable is [28]

$$\xi = \frac{2x}{1 + \sqrt{1 + 4y^2 x^2}}. \tag{14}$$

Clearly, the two Nachtmann moments are simultaneously constructed by using the two spin structure functions  $g_{1,2}$ . According to eqs. (5-6), one can expand the two Nachtmann moments up to the order of  $M^6/Q^6$  as

$$\begin{aligned}
M_1^{(n)} &= a_n + \mathcal{O}(y^8) \\
M_2^{(n)} &= d_n + \mathcal{O}(y^8).
\end{aligned} \tag{15}$$

Eq. (15) explicitly tells that, different from eqs. (1) and (11), one can get the contributions of the pure twist-2 with spin- $n$  and the pure twist-3 with spin- $(n-1)$  operators from the Nachtmann moments. The above conclusion is valid when the target mass corrections are included. If the target mass corrections vanish, then eq. (11) turns to be  $\tilde{d}_2$ . The advantage of the Nachtmann moments, then, means that they contain only dynamical

higher-twist effects, which are the ones related to the correlations among the partons. As a result, one sees that the Nachtmann moments are constructed to protect the moments of the nucleon structure functions from the target mass corrections. It is clear that the Nachtmann moments relate to the dynamical matrix elements of the operators with the definite twist and definite spin [17,21,24].

### 3 The numerical estimate of the target mass corrections

One may estimate the target mass corrections to the contribution of the twist-3  $\tilde{d}_2 = 2d_3$ . We note that the nucleon spin structure functions are

$$\begin{aligned} g_1(x, Q^2) &= \frac{F_2(x, Q^2)[A_1(x, Q^2) + \gamma A_2(x, Q^2)]}{2x[1 + R_\sigma(x, Q^2)]}, \\ g_2(x, Q^2) &= \frac{F_2(x, Q^2)[-A_1(x, Q^2) + A_2(x, Q^2)/\gamma]}{2x[1 + R_\sigma(x, Q^2)]}, \end{aligned} \quad (16)$$

where  $F_2(x, Q^2)$  is the unpolarized structure function,  $\gamma^2 = 4y^2x^2$ , and  $R_\sigma(x, Q^2) = \sigma_L(x, Q^2)/\sigma_T(x, Q^2)$  is the ratio of the longitudinal to transverse virtual photon cross sections. In eq. (16),  $A_{1,2}(x, Q^2)$  are the virtual photon asymmetry parameters. To analyze the target mass corrections to the contribution of the twist-3  $\tilde{d}_2 = 2d_3$  numerically, we simply employ the empirical parameterization forms of  $g_1^{p,d}$  [16,18], of  $R_\sigma(x, Q^2)$  [32], of  $F_2^{p,d}$  [33], and of the average data of  $g_2^{p,d}(x, Q^2)$  [20]. Here, we stress that the well-known Wandzura and Wilczek (WW) relation [34] for  $g_2$

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy \quad (17)$$

is valid [30-31] if only the leading-twist is considered. The target mass corrections to the twist-2 contribution do not break the WW relation. If the higher-twist operators, like twist-3, are considered, the WW relation  $g_2(x, Q^2) = g_2^{WW}(x, Q^2)$  is no longer valid. Thus, according to Ref. [20], one may write

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2). \quad (18)$$

Clearly, if only the twist-2 term is considered,  $\bar{g}_2(x, Q^2) = 0$ . The non-vanishing value of  $\bar{g}_2$  results from the higher-twist effects. In Ref. [20], the average data of  $A_2(x_0, Q_0^2)$  and  $xg_2^{p,d}(x_0, Q_0^2)$  are listed for the measured values of  $x_0$  and  $Q_0^2$ . The measured ranges of the experiment are  $0.021 \leq x_0 \leq 0.790$  and  $0.80 \text{ GeV}^2 \leq Q_0^2 \leq 8.20 \text{ GeV}^2$ . It is argued

that  $\bar{g}_2(x, Q^2)$  is independent of  $Q^2$  in the measured region since  $\tilde{d}_n$  depends only logarithmically on  $Q^2$  [5]. Thus, the  $Q^2$ -dependence of the structure function  $g_2(x, Q^2)$  mainly results from  $g_2^{WW}(x, Q^2)$  [20]. Another point made by E155 collaboration [20] is that the contributions of  $\bar{g}_2$  to the integral of eq. (1) beyond the measured  $x$  region, say  $x < 0.021$  and  $x > 0.790$ , are negligible.

Based on the above two arguments of Ref. [20], we calculate the ratio  $R(Q^2)$  of the Nachtmann moment to the value of eq. (1)

$$R(Q^2) = \frac{2M_2^{(3)}(Q^2)}{I(Q^2)} \quad (19)$$

for the proton and neutron spin structure functions, respectively. For the neutron structure function, we simply use the relation

$$g_1^n(x, Q^2) = \frac{2}{1 - \omega_D} g_1^d(x, Q^2) - g_1^p(x, Q^2), \quad (20)$$

with  $\omega_D \sim 0.05$  being the deuteron D-state probability. It should be mentioned that the target mass corrections to the deuteron (spin one) structure functions have been discussed in Ref. [35]. It is found that the target mass corrections to  $F_{1,2}^d$  and  $g_{1,2}^d$  are precisely the same as in the spin-1/2 targets. The calculated results of  $R(Q^2)$  are plotted in Fig. 1. Clearly, the divergence of the two ratios from unity displays the target mass corrections. From Fig. 1 we see a sizeable role played by the target mass corrections. When the momentum transfer  $Q^2$  becomes small, the role increases obviously. The effect of the target mass corrections is about 30% for the average of the proton and neutron targets when  $Q^2 \sim 1.6 \text{ GeV}^2$ . It decreases to about 10% when  $Q^2 \sim 8 \text{ GeV}^2$ . We find that the values of the Nachtmann moment  $2M_2^{(3)}$ , which contains no target mass corrections, are always smaller than those of  $I(Q^2)$  which has the target mass corrections. Therefore, we conclude that the contribution of the twist-3  $\tilde{d}_2$  to the nucleon structure function extracted from eq. (1) is overestimated. For example, at an average  $Q^2$  of  $5 \text{ GeV}^2$  of the E155 experiment [20], the estimated  $\tilde{d}_2^p = 0.0025 \pm 0.0016 \pm 0.0010$  from experiment data and eq. (1). If the target mass corrections are taken into account, the central value of  $\tilde{d}_2^p$  is reduced to about 0.00215. This new value becomes even close to zero. Moreover, Fig. 1 shows that the ratio of the neutron is similar to that of the proton, and the target mass corrections to the neutron is only a little bit smaller than those to the proton. In addition, the uncertainties of  $\tilde{d}_2$  in Ref. [20] come from the statistical and systematic errors on measured  $xg_2$ . At a low  $Q^2$  value, say,  $Q^2 = 1 \text{ GeV}^2$ , the measured  $xg_2^p$  is  $0.010 \pm 0.007$ . When  $Q^2$  increases to about  $8 \text{ GeV}^2$ , the measured  $xg_2^p = -0.007 \pm 0.002$ . Clearly, the errors on  $\tilde{d}_2$  caused by

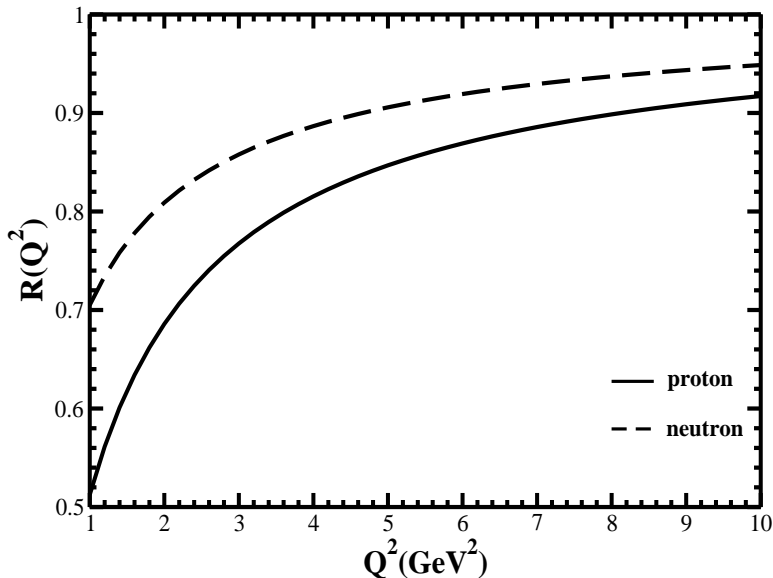


Figure 1: Ratio  $R(Q^2)$ . The solid and dashed curves are the results of the proton and neutron, respectively.

the uncertainties of  $xg_2$  are much larger than the target mass corrections in our calculation.

## 4 Conclusions

In summary, we have explicitly shown the target mass corrections to the contribution of twist-3 extracted from eq. (1). It is clear that the Nachtmann moments are constructed by the two spin structure functions and do not governed by the target mass corrections. Therefore, in order to precisely extract the contribution of the twist-3  $\tilde{d}_2$  at different  $Q^2$  values, one is required to employ the Nachtmann moment  $2M_2^{(3)}$ . The numerical estimate ratios  $R(Q^2)$  for the proton and neutron in Fig. 1 indicate that the target mass corrections play a sizeable role to the integral of eq. (1). Thus, the equation gives the result which is different from the pure contribution of the twist-3  $\tilde{d}_2$ . Moreover, the values of  $\tilde{d}_2$  from eq. (1) are always overestimated because  $I(Q^2)$  contains the mixed contributions of other twist-3 terms with higher-spin, except for  $d_3$  (or  $\tilde{d}_2$ ). Numerically, the effect of the target mass corrections in the extracted  $\tilde{d}_2$  (or  $d_3$ ) from eq. (1) is about 30% for the average of the proton and neutron targets when  $Q^2 \sim 1.6 \text{ GeV}^2$  or about 10% when  $Q^2 \sim 8 \text{ GeV}^2$ . The corrections to the neutron are similar to those to the proton. As a result, we ex-



pect that one should take the Nachtmann moment to precisely extract the contribution of twist-3 operator  $\tilde{d}_2$ . With a very accurate value of  $\tilde{d}_2$ , one may further get the precise information of the other higher-twist operators, like twist-4 term  $\tilde{f}_4$ . Since the errors of the present data of  $xg_2^{p,d}$  are large, the uncertainties of the extracted values of  $\tilde{d}_2^{p,d}$  from eq. (1) are larger than the target mass corrections. A much more precise measurement of  $g_2$  and  $A_2$  is urgently required.

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